Network Theory and Brain network

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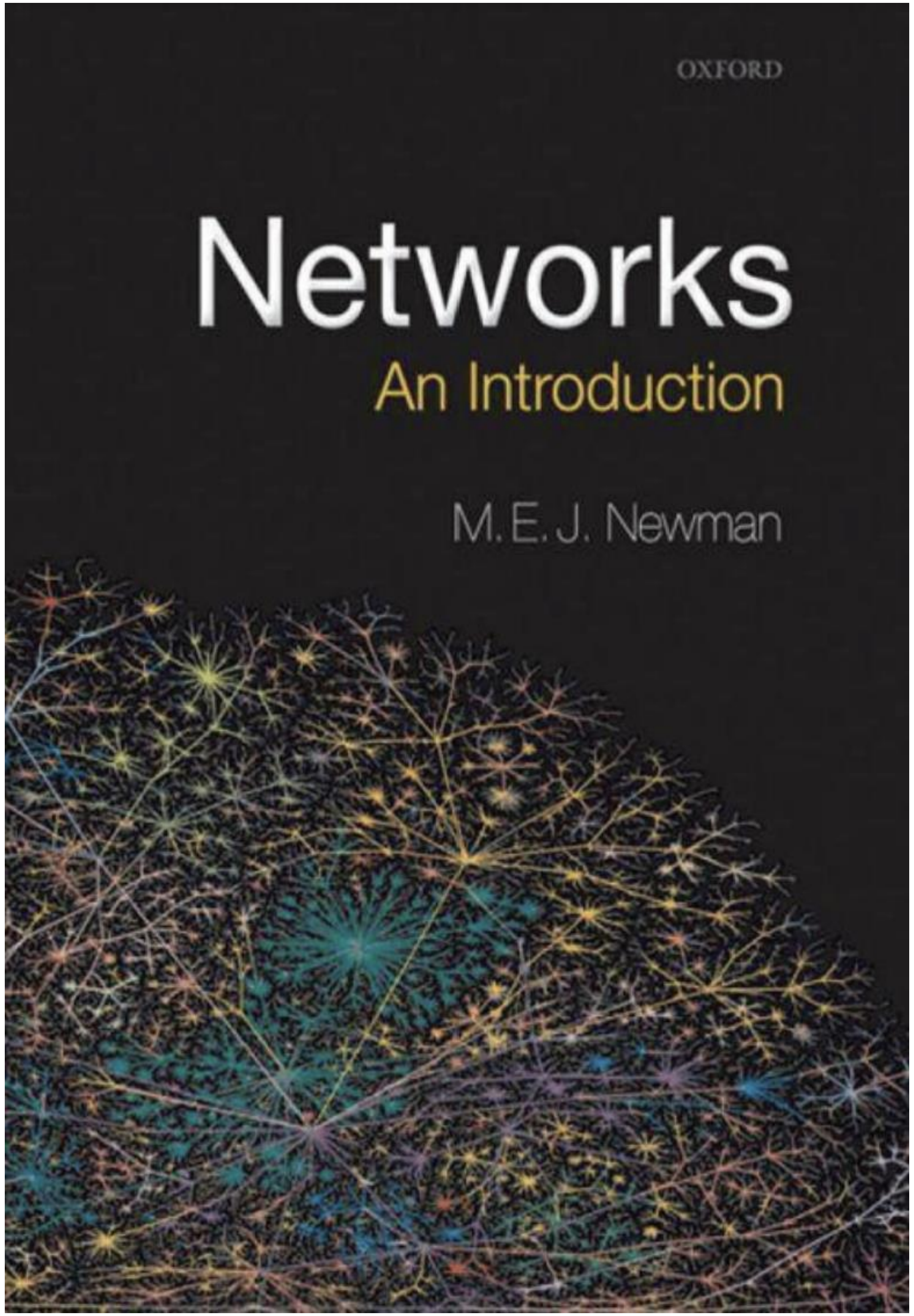
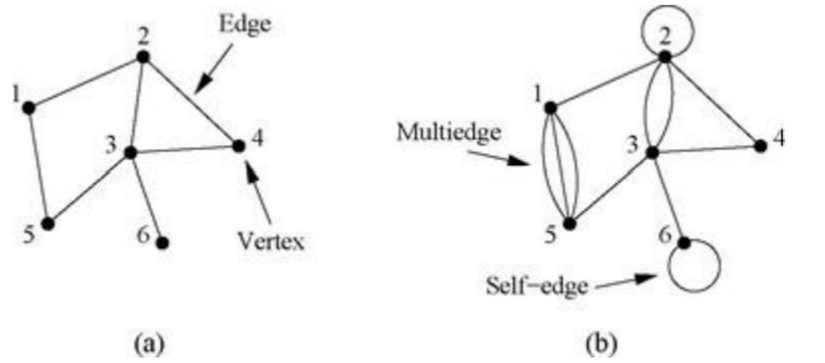
2023-11-21

Simple network theory


OXFORD
Networks
An Introduction
M. E. J. Newman

Adjacency matrix

- $A_{ij} = \begin{cases} 1 & i \text{ and } j \text{ connected} \\ 0 & \text{otherwise} \end{cases}$

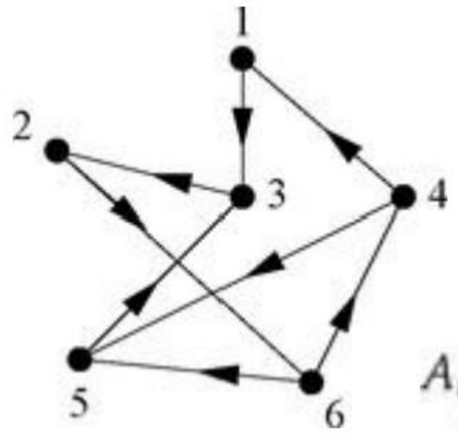


$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}, \qquad \mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 3 & 0 \\ 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 \end{pmatrix}$$

Various networks

weighted matrix

Direction matrix

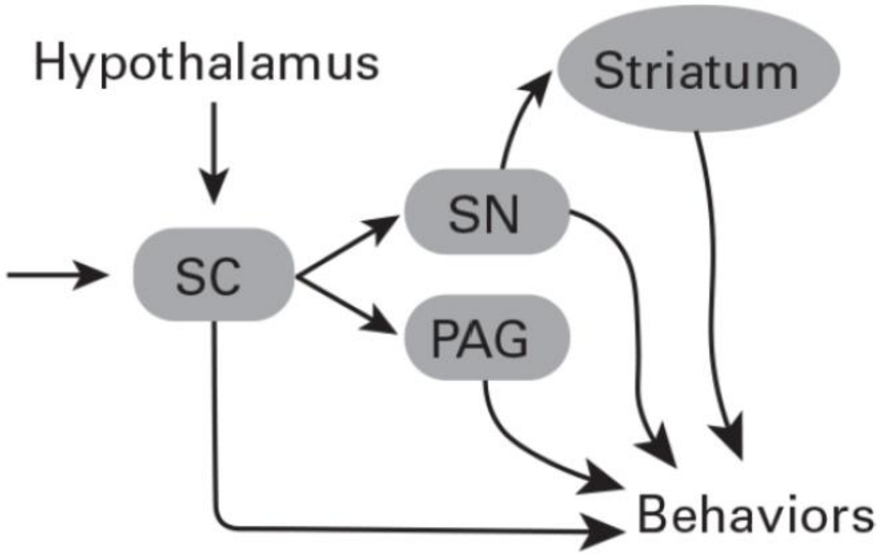


$$A_{ij} = \begin{cases} 1 & \text{if there is an edge } \textit{from } j \textit{ to } i, \\ 0 & \text{otherwise.} \end{cases}$$

$$\mathbf{A} = \begin{pmatrix} 0 & 2 & 1 \\ 2 & 0 & 0.5 \\ 1 & 0.5 & 0 \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Various networks



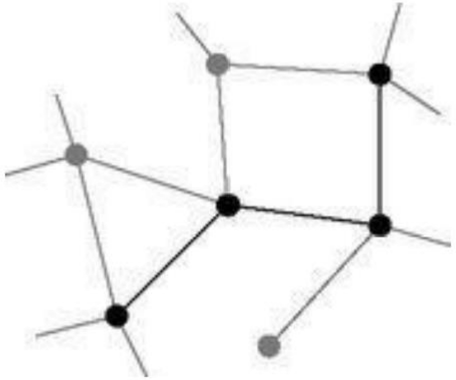
Valence; regulation
context; motivation;
sensory

Degree

- Number of edges connected to one vertex.
- Undirected: $m = \frac{1}{2}\sum_{i,j} A_{ij}$
- Direction이 존재한다면?

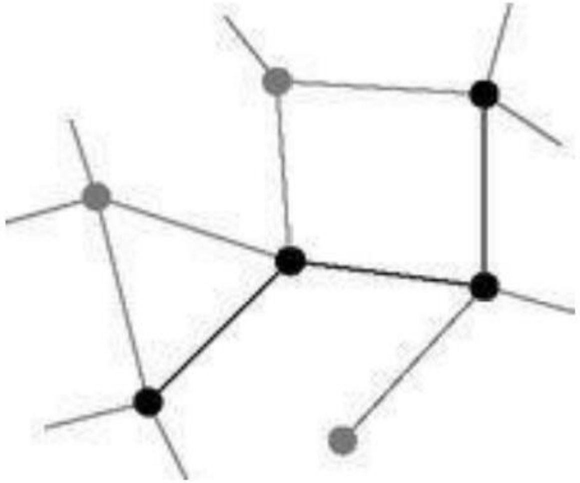
$$A_{ij} = \begin{cases} 1 & \text{if there is an edge } \textit{from } j \textit{ to } i, \\ 0 & \text{otherwise.} \end{cases}$$

$$k_i^{in} = \sum_{j=1}^{n} A_{ij} \qquad k_j^{in} = \sum_{i=1}^{n} A_{ij}$$



Path

- any sequence of vertices such that every consecutive pair of vertices in the sequence is connected by an edge in the network



Network Centrality

Degree centrality

- "Which are the most important or central vertices in a network?" → centrality
- Degree centrality: Which has the greatest number of degree(in-degree, out-degree)

The physics of brain network structure, function and control

Christopher W. Lynn & Danielle S. Bassett

Nature Reviews Physics **1**, 318–332 (2019) | Cite this article

11k Accesses | **162** Citations | **205** Altmetric | Metrics

Betweenness centrality

- measures the extent to which a vertex lies on minimum paths between other vertices
- $maximum\ betweeness = \frac{N(N-1)}{2}$

Closeness centrality

- measures the mean distance from a vertex to other vertices
- $Closeness\ centrality = \frac{1}{\frac{1}{N-1}\sum L_{X,I}}$

Eigenvalue centrality

- the value of connecting is important!
- Let A is adjacency matrix

$$x_i' = \sum_j A_{ij} x_j, \qquad \mathbf{x}(t) = \mathbf{A}^t \mathbf{x}(0). \qquad x(0) = \sum c_i v_i$$

i번째 연결되어 있는 node의 centrality i번째 노드의 centrality를 변화시킨다.
$t \rightarrow \infty$일 때 수렴하는 x 존재하는가?

$$\mathbf{x}(t) = \mathbf{A}^t \sum_i c_i \mathbf{v}_i = \sum_i c_i \kappa_i^t \mathbf{v}_i = \kappa_1^t \sum_i c_i \left[\frac{\kappa_i}{\kappa_1}\right]^t \mathbf{v}_i,$$

κ_1 maximum absolute value eigenvalue

Eigenvalue centrality

$$\mathbf{x}(t) = \mathbf{A}^t \sum_i c_i \mathbf{v}_i = \sum_i c_i \kappa_i^t \mathbf{v}_i = \kappa_1^t \sum_i c_i \left[\frac{\kappa_i}{\kappa_1}\right]^t \mathbf{v}_i,$$

κ_1 maximum absolute eigenvalue

- $\lim_{\substack{t \to \infty \\ i \neq 1}} \left(\frac{\kappa_i}{\kappa_1}\right)^t = 0$

$$\mathbf{A}\mathbf{x} = \kappa_1 \mathbf{x}.$$

$$x_i = \kappa_1^{-1} \sum_j A_{ij} x_j,$$

Katz centrality

- Zero-in degree node: centrality 0

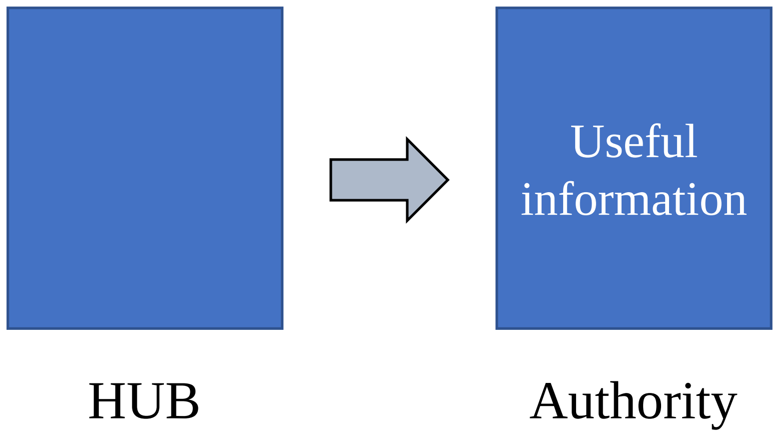
$$x_i = \alpha \sum_j A_{ij} x_j + \beta,$$

- once they have a non-zero centrality, then the vertices they point to derive some advantage from being pointed to

$$\mathbf{x} = (\mathbf{I} - \alpha \mathbf{A})^{-1} \boldsymbol{\beta},$$

Network Hub

- Authorities: nodes that contain useful information on a topic of interest
- Hubs: nodes that tell us where the best authorities are to be found



Hub centrality $x_i = \sum_j A_{ij} y_j$

Authority centrality $y_i = \sum_j A_{ji} x_j$

Network Hub

- 뉴런의 네트워크의 전체적인 특성을 분석하기 위해 (어떤 뉴런 또는 어떤 영역) centrality나 hub를 분석할 수 있음.

> Int J Neural Syst. 2018 Mar;28(2):1750013. doi: 10.1142/S0129065717500137. Epub 2016 Nov 16.

From Structure to Activity: Using Centrality Measures to Predict Neuronal Activity

Jack McKay Fletcher [1], Thomas Wennekers [1]

Affiliations + expand

PMID: 28076982 DOI: 10.1142/S0129065717500137

Random graph

- the network in which we fix only the number of vertices n and the number of edges m
- Let the probability to make connectivity is p
- Probability of making random graph $G(m, p)$ $P(G) = p^m(1-p)^{\binom{n}{2}-m},$
- Mean value of m $\langle m \rangle = \sum_{m=0}^{\binom{n}{2}} mP(m) = \binom{n}{2}p.$
- Probability to have one vertex have k connect
- $p_k = \binom{n-1}{k} p^k (1-p)^{n-1-k}$

Random graph

- Improved version

If node have large connectivity → probability to have new connecting increasing

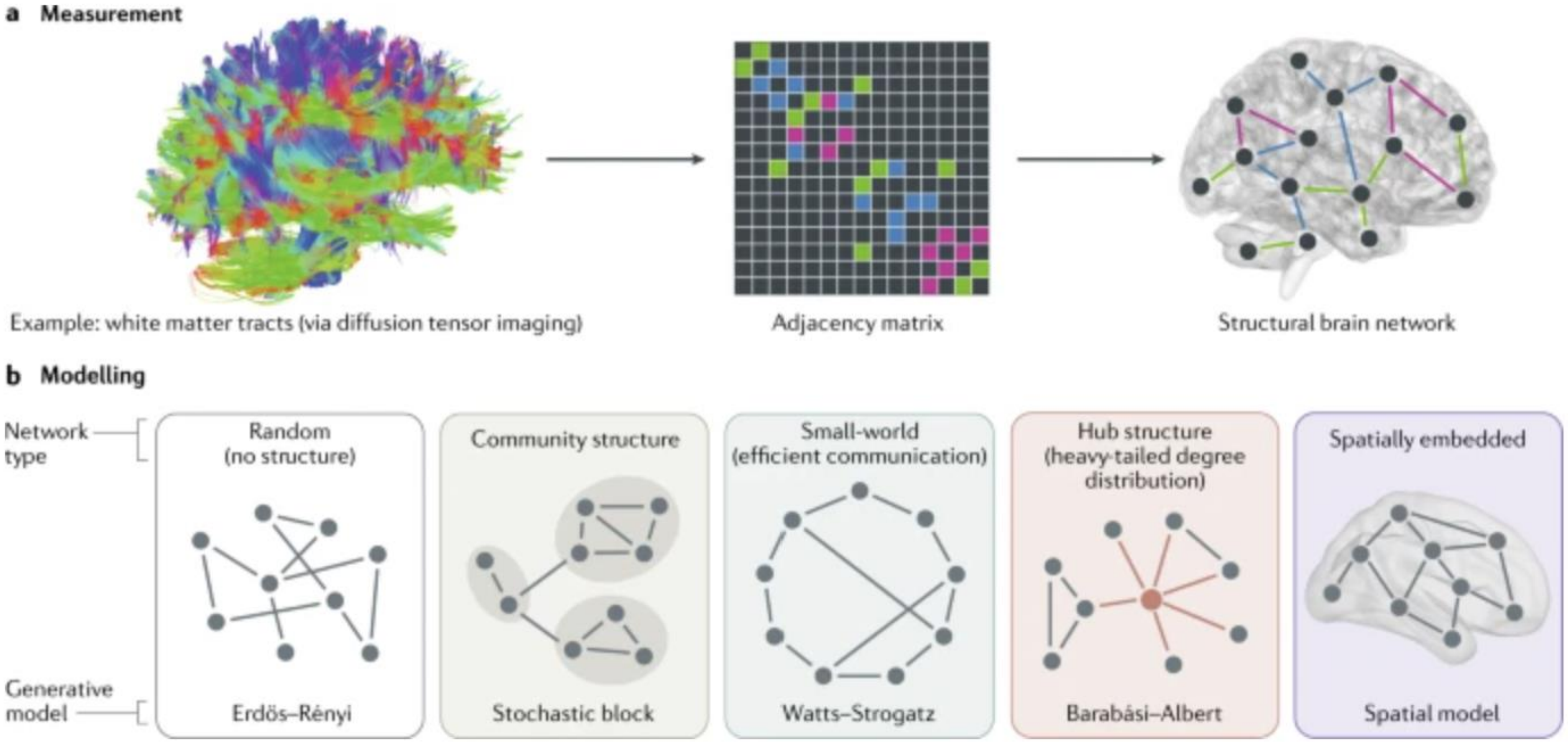
: Babarasi – Albert model

Each node have fitness: a quantitative measure of a node's ability to stay in front of the competition → fitness increasing, probability to have new connecting increasing

: Bianconi – Babarasi model

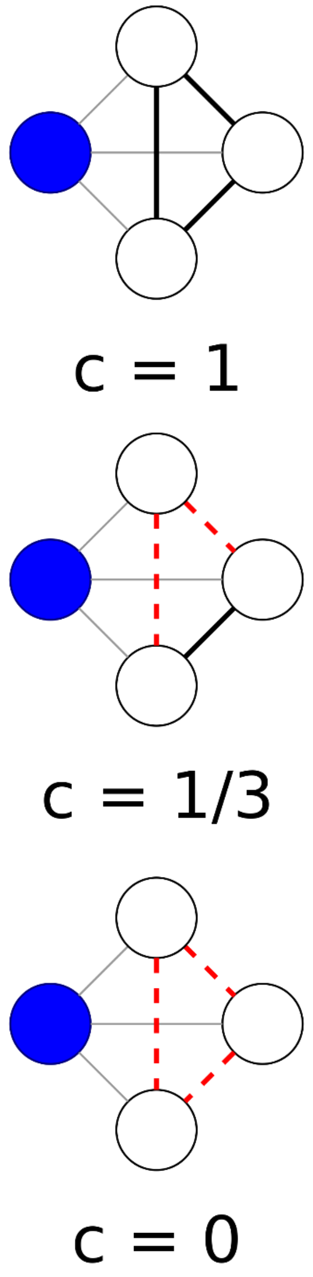
Expecting the growth of network

Random graph



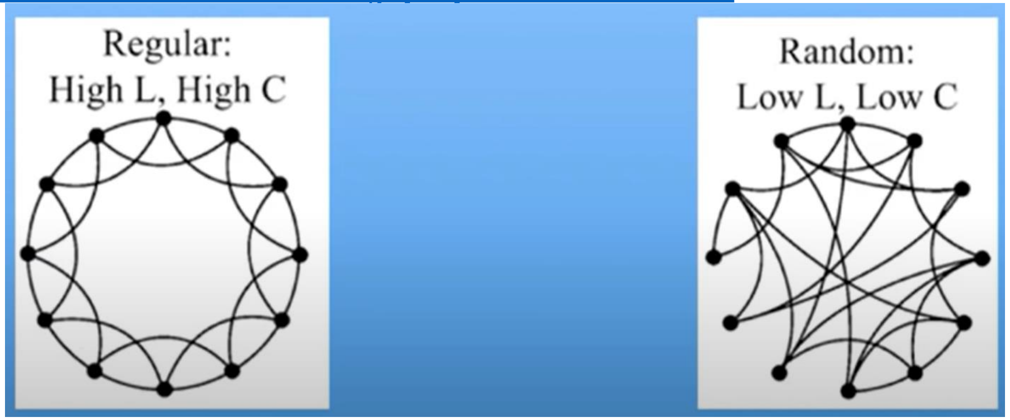
Clustering coefficent

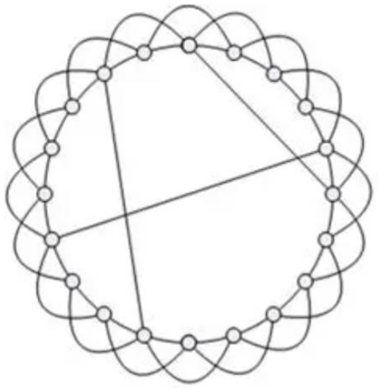
- measure of the degree to which nodes in a graph tend to cluster together
- 무리 지으려고 하는 경향성



Small world

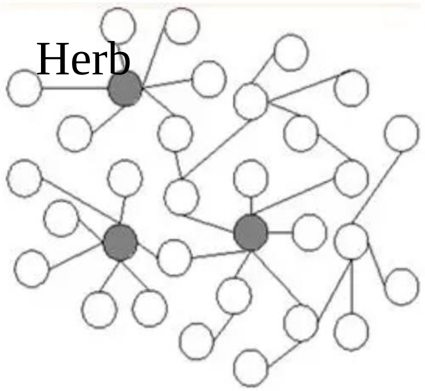
- https://www.youtube.com/watch?v=ypqrwiR0GiA





Small-world network

Duncan Watts and Steven Strogatz



Scale-free network

바바라시

Small world

- Efficiency: the effectiveness of information spread across members of a network
- systems made of locally clustered nodes + a *small* number of random connections → *all* nodes to be accessible within a small number of connectivity steps
- signals can influence distal elements of a system even when physical connections are fairly sparse.

Brain network

Intro

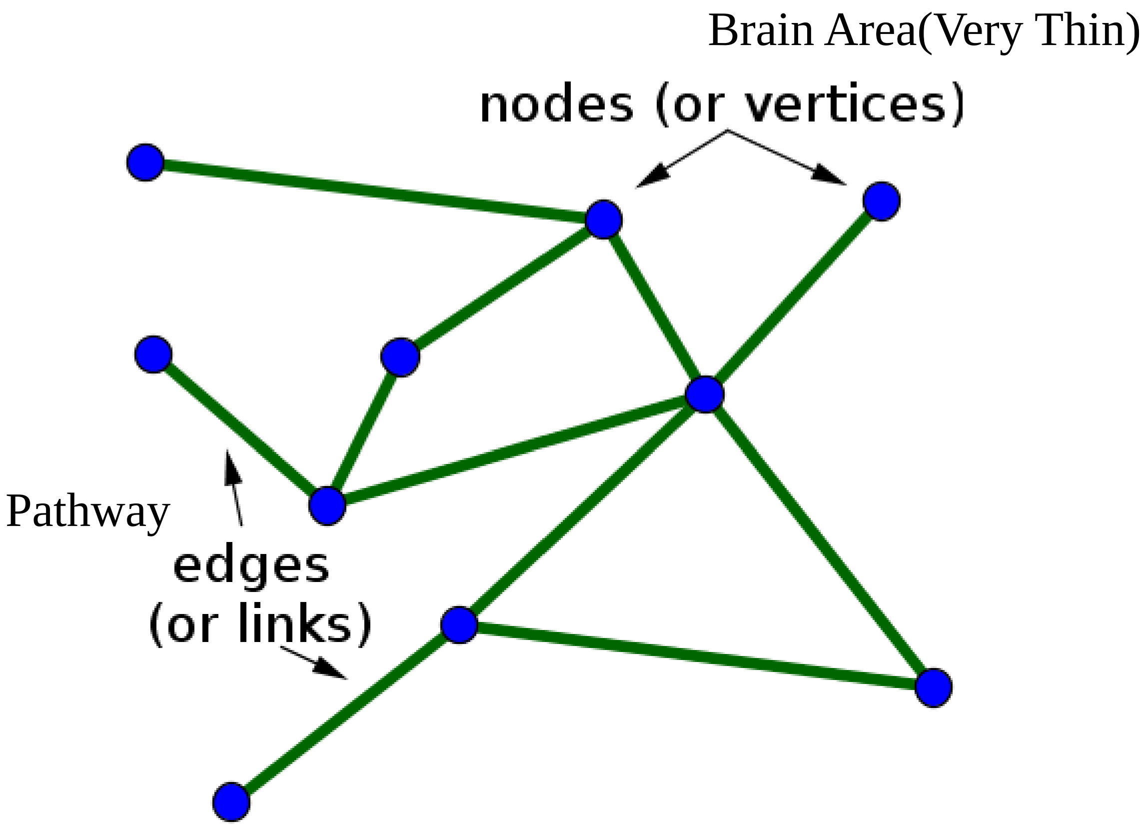
- Large-scale brain circuits → functionally integrated systems

1) The brains anatomical and functional architecture → highly non-modular

2) Functional architecture → highly dynamic, context-sensitive

1. Massive Combinatorial Anatomical Connectivity

- Research for pathway crossing the brain (anatomical) → uncomplete
- Massive connectivity



1. Massive Combinatorial Anatomical Connectivity

- Brain is more interconnected than small world
- So, while it is true that local connectivity predominates within the cortex, there are enough medium- and long-range connections—in fact, more than the “minimum” required—for information to spread around remarkably well.

1. Massive Combinatorial Anatomical Connectivity

- 200 anatomical tracing studies of the macaque brain

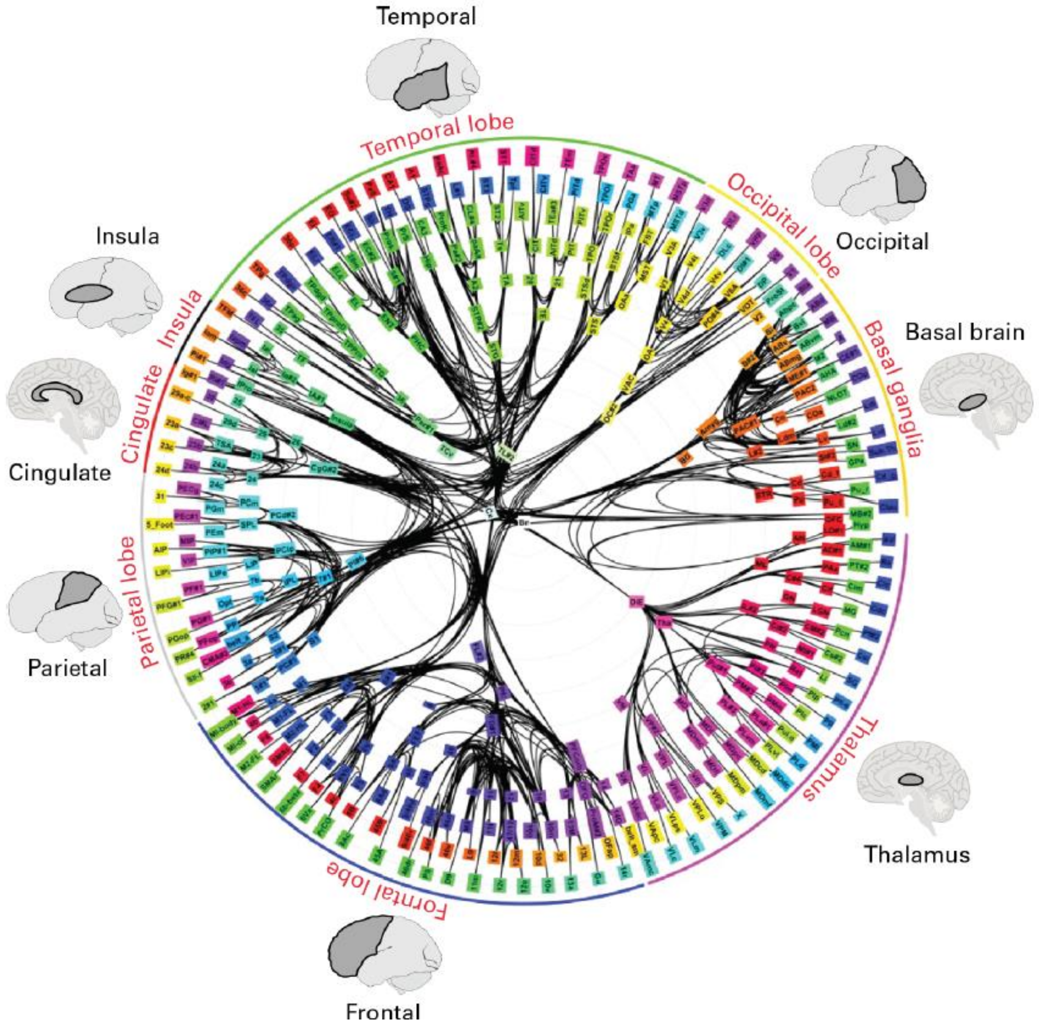
1) It is a set of regions that is far more tightly integrated than the overall brain

2) information likely spreads more swiftly within the core than through the overall brain

3) brain communication relies heavily on signals being communicated via the core

1. Massive Combinatorial Anatomical Connectivity

- monkey cortex
- 17 heavily interconnected brain regions
- across the parietal, temporal, and frontal cortex.
- “rich club” exists

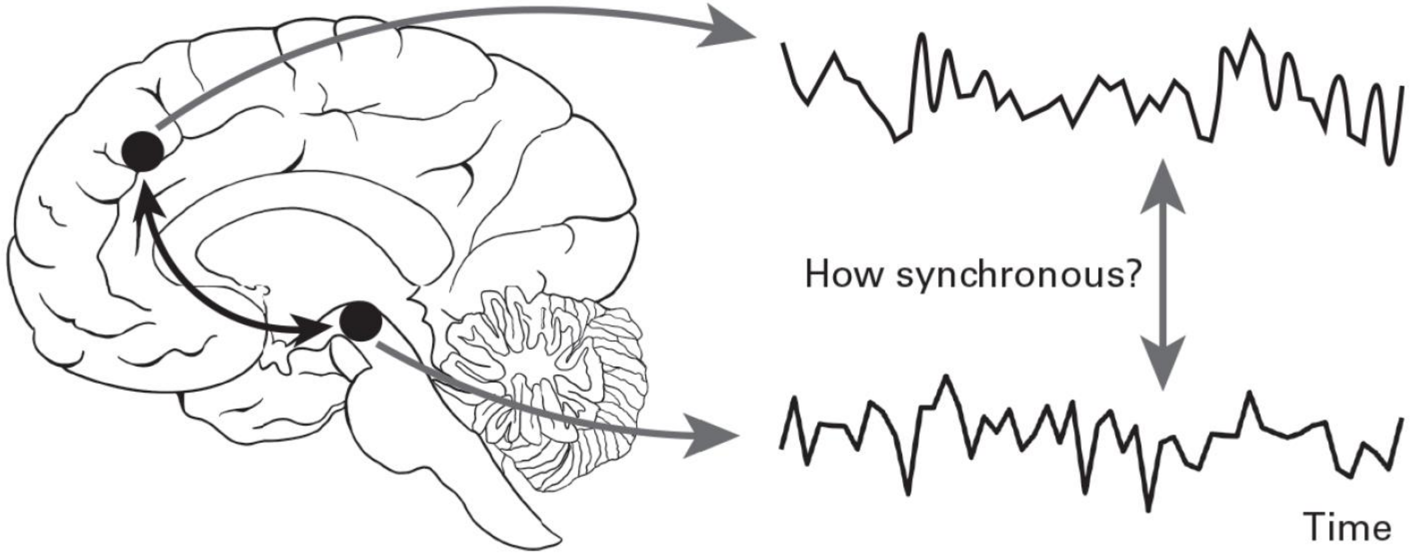


2. High Distributed Functional Connectivity

- *functional connection* between genes.
- Protein -> functionally related. Gene -> *functional connection*.
- influence the probability of responding at B / they actually cause cells in B to fire
- connections between regions are not simply binary / single weight

2. High Distributed Functional Connectivity

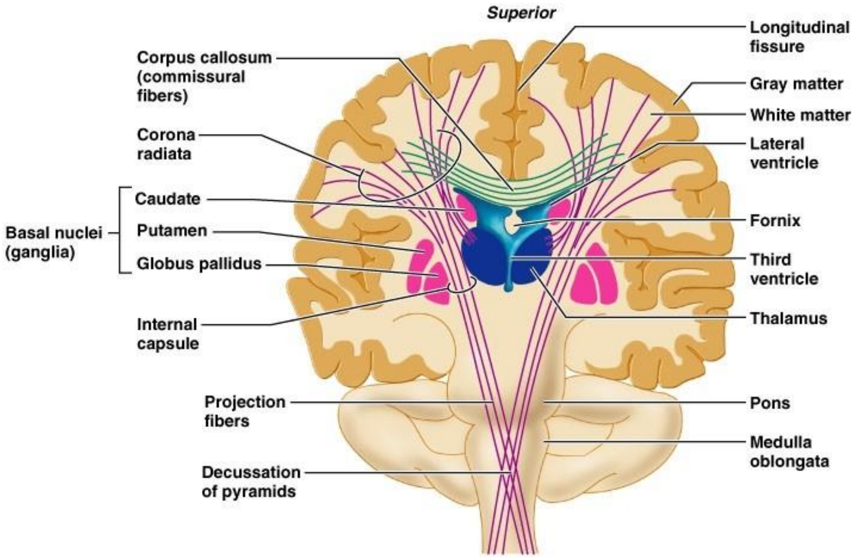
- Functional connectivity measures the extent to which signals from two regions are in synchrony.
- Whether or not the regions are directly connected by an anatomical pathway is unimportant. → easy



2. High Distributed Functional Connectivity

- illustrated by adults born without the corpus callosum, which contains massive bundles of axonal extensions joining the two hemispheres. Although starkly different structurally relative to controls, individuals without the callosum exhibit very similar patterns of functional connectivity compared to normal individuals

- Functional connectivity should be investigated.

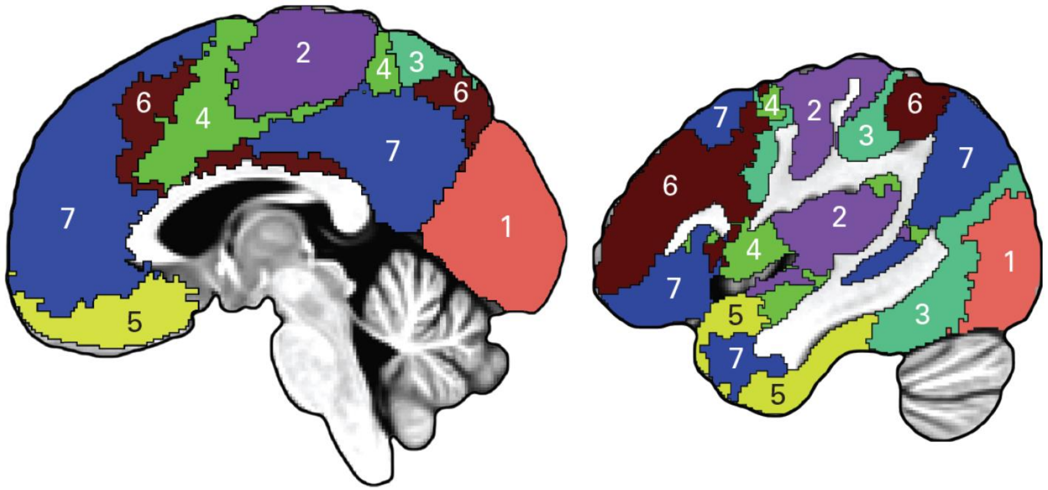


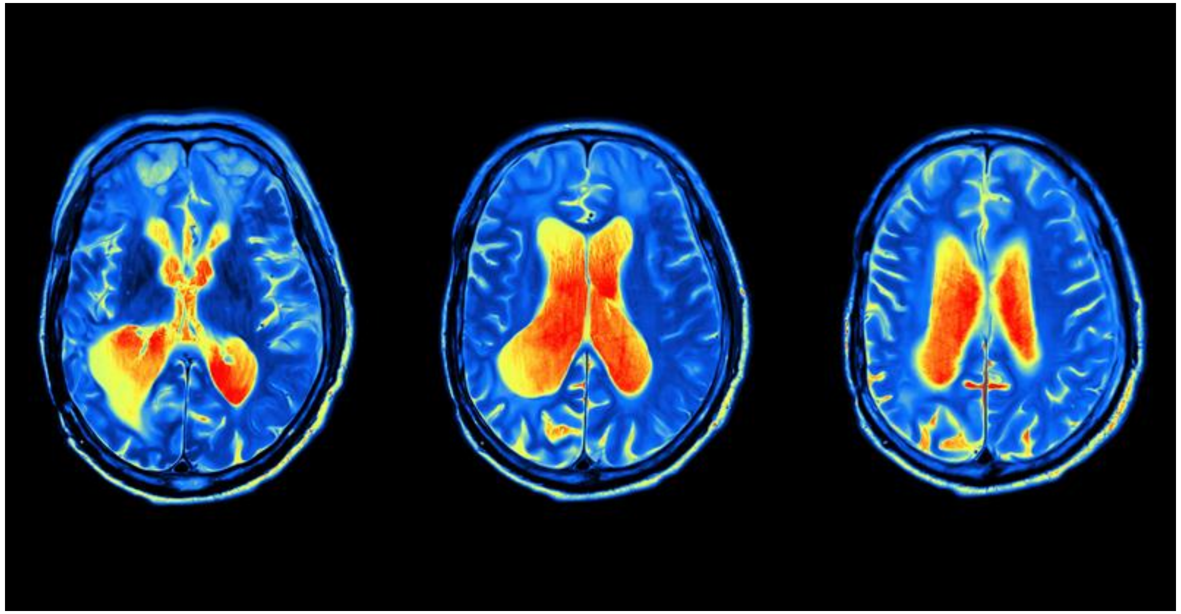
3. Networks as functional unit

- *The network itself is the unit*, not the brain area → dynamic..!
- *cell assembly:* strongly interconnected group of active neurons (Donald Hebb)

3. Networks as functional unit

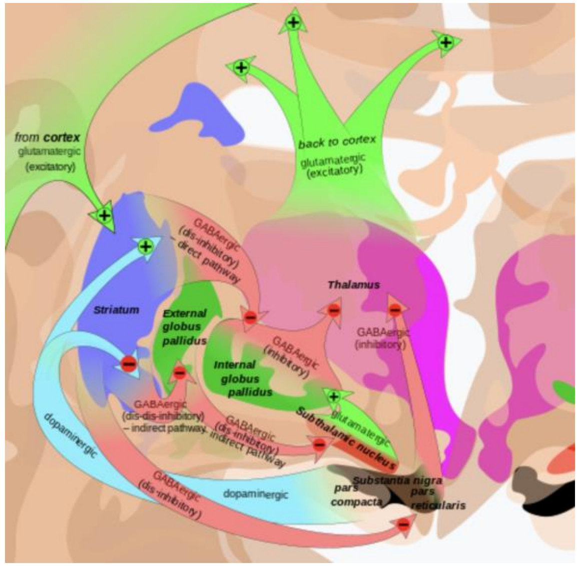
- fMRI -> existence of large-scale networks
- “seed analysis” -> determine the extent to which each brain location is synchronized with the seed (functional network)

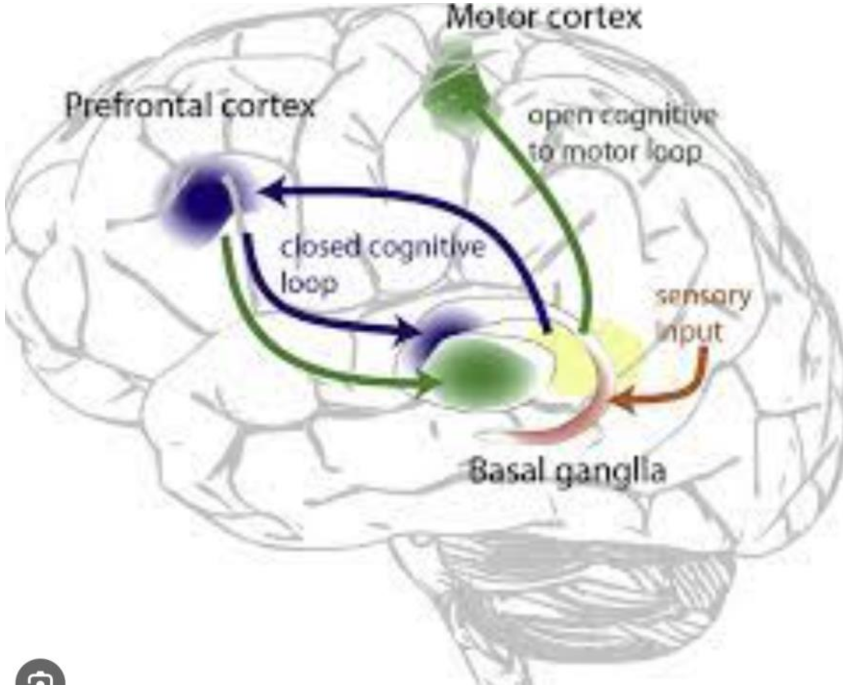




4. Interactions via Cortical-Subcortical Loops

- If we think of connectivity along the cortex as "horizontal," a cortico-centric standpoint misses the "vertical" (cortical-subcortical) features of anatomical pathways.
- one small piece, all of the cortex projects to the striatum
- "cortico-basal ganglia-thalamo-cortical" loops



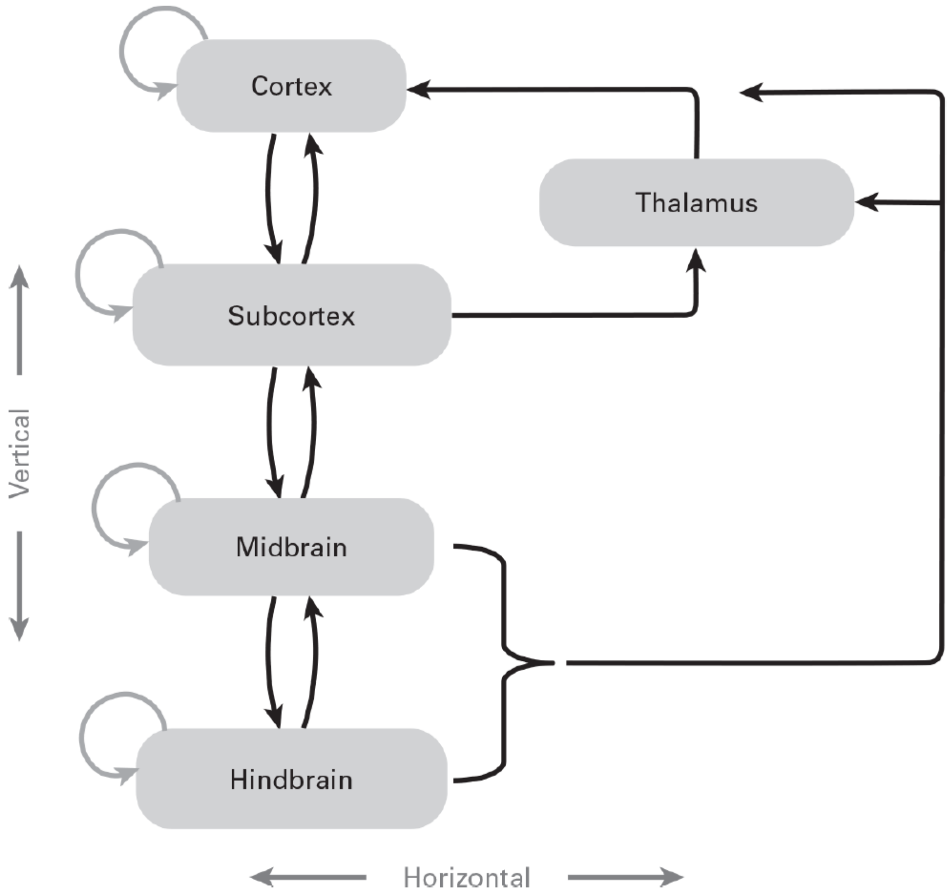


5. Connectivity with the Body

- the central nervous system is in constant two-way communication with the body on a much broader scale through a process called interoception that is both varied and nuanced, encompassing feelings related to muscular and visceral sensations, pain, and itch, among many others.

Networks Are Dynamic

- clustering methods
- community should have more internal than external associations
- unique groupings(most popular) vs multiple network
- “airport hub”
- That’s when they are viewed not as fixed collections of regions but instead as coalitions that form and dissolve to meet computational needs


Cortex
Thalamus
Subcortex
Vertical
Midbrain
Hindbrain
Horizontal

Network Analysis with python

```
import networkx as nx
Import matplotlib.pyplot as plt

#객체 생성
g1 = nx.DiGraph()  # 무방향성 그래프의 경우, nx.Graph

#노드 추가
g1.add_node(4)
g1.add_node(1)
g1.add_node(2)
g1.add_node(3)

#노드 제거
g1.remove_node(3)
g1.add_node(3)
```

Network Analysis with python

```
#엣지 추가
g1.add_edges_from([(1, 2), (1, 3), (2, 4), (3, 4)])

#centrality 계산
btw = nx.betweenness_centrality(g1, normalized=True, endpoints=True)
clo = nx.closeness_centrality(g1)
eigen = nx.eigenvector_centrality(g1)
degree = nx.degree_centrality(g1)
od = nx.out_degree_centrality(g1)
id = nx.in_degree_centrality(g1)

print(btw,clo,eigen,degree,od,id)
```

Network Analysis with python

```
#노드 사이즈,  centrality에 따라 다르게 해서 그리기

node_color=[500.0*v for v in clo]
node_size=[v*7000 for v in btw]
plt.figure(figsize=(20,20))

nx.draw(g1, node_color=node_color, node_size=node_size, with_labels = True, font_size = 8)

plt.show()

plt.axis('off')
```